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On the Unit Groups of Burnside Rings.

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1. Introduction.

Let G be a finite group and let $\underline{\underline{Set}}_f{}^G$ be the category of finite (right) G-sets and G-maps. The Grothendieck ring of this category (with respect to coproduct $+$ and product $\times$) is called the Burnside ring of G and is denoted by $A(G)$.

A super class function is a map of the set of subgroups of G to $\underline{\underline{Z}}$ which is constant on each conjugate class of subgroups. Let $\widetilde{A}(G) = \underline{\underline{Z}}^{Cl(G)}$ be the ring of super class functions. For any subgroup S of G, the map $[X] \longrightarrow |X^S|$ extends to a ring homomorphism $\varphi_S : A(G) \longrightarrow \underline{\underline{Z}}$, and so we have a ring homomorphism

$$\varphi = \prod_{(S)} \varphi_S : A(G) \longrightarrow \widetilde{A}(G) = \underline{\underline{Z}}^{Cl(G)}.$$

This map is injective. Thus we can identify any element x of $A(G)$ with the super class function $\varphi(x)$, and so we simply write $x(S) := \varphi(x)(S) = \varphi_S(x)$ for a subgroup S. See tom Dieck [Di79] Chapter 1.

Now, by geometric methods, tom Dieck proved that for an $\mathbb{R}G$-module V, the function

$$u(V) : S \longmapsto \operatorname{sgn} \dim V^S$$

belongs to the Burnside ring $A(G)$, where $\operatorname{sgn} n := (-1)^n$ ([Di79] Proposition 5.5.9). The first purpose of this paper is to prove this fact by purely algebraic methods. In fact we shall prove the following theorem in Section 2.

Theorem A. *Let G be a finite group and let V be a $\mathbb{C}G$-module with real valued character. Then the function*

$$u(V) : S \longmapsto \operatorname{sgn} \dim_{\mathbb{C}} V^S$$

is a member of the Burnside ring $A(G)$.

Since clearly $u(V)^2 = 1$ and $u(V \oplus W) = u(V)\, u(W)$, we have a group homomorphism into the unit group :

$$u = u_G : \overline{R}(G) \longrightarrow A(G)^*,$$

where $\overline{R}(G)$ is the ring of real valued virtual characters of G. We call this map u_G a tom Dieck homomorphism.

There are various maps between Burnside rings (and unit groups), and the assignment $A^* : H \longmapsto A(H)^*$ together with restrictions and multiplicative inductions forms a so called G-functor (= a Mackey functor from the category of finite G-sets) and further that the tom Dieck homomorphism gives a morphism between G-functors. Since A^* is a G-functor, we have that $A(G)^*$ is a module over $A(G)$ (and also over $A(G)_{(2)}$, the localization at 2). In fact, the action $A(G)^* \times A(G) \longrightarrow A(G)^*$ is induced by the exponentaial map $(Y, X) \longrightarrow Y^X$ (the set of all maps of X to Y). From the theory of Burnside rings and G-functors, we can show some transfer theorems about $A(G)^*$. See Section 3. The proof will appear in another paper.

Notation and terminology. We always denote by G a finite group. The set of G-conjugate classes (H) of subgroups H of G is denoted by $Cl(G)$. For subsets A, B of G, we mean by $A =_G B$ (resp. $A \le_G B$) that A and B are conjugate in G (resp. A is G-conjugate to a subgroup of B). We put $A^g := g^{-1}Ag$. When a group G acts on a set X, we denote by X^G the set of elements fixed by G. The ordinary character ring of G is denoted by $R(G)$. For a ring R, the unit group of R is denoted by R^*. The inner product of characters χ and θ is denoted by $<\chi, \theta>$. Other notation and terminology are standard. See [Go68], [Di79].

2. Proof of Theorem A.

In this section, we prove Theorem A. As in the introduction, let G be a finite group and let $Cl(G)$ denote the set of conjugate classes of subgroups of G. We mean by (S) the class of a subgroup S. We set $WS := N_G(S)/S$ for a subgroup S of G.

Lemma 2.1. *There is an exact sequence of abelian groups:*

$$0 \longrightarrow A(G) \xrightarrow{\varphi} \mathbb{Z}^{Cl(G)} \xrightarrow{\psi} \prod_{(S)} (\mathbb{Z}/|WS|\mathbb{Z}) \longrightarrow 0,$$

where φ *is the injective ring homomorphism given in the introduction, and for a super class function* x, *the* S-*component of* $\psi(x)$ *is defined by*

$$(x)_S := \sum_{gS \in WS} x(<g>S) \quad \bmod |WS|.$$

This lemma is well-known and its proof is found in, for example, tom Dieck [Di79] 1.3. We can now prove Theorem A.

Proof of Theorem A. Let χ be the real valued character afforded by the $\mathbb{C}G$-module V, and let $u(V)$ be the present super class function:

$$u(V) : (S) \longmapsto \operatorname{sgn} \dim V^S.$$

By Lemma 2.1, we must show that for each subgroup S of G,

(1) $$\sum_{gS \in WS} u(V)(\langle g \rangle S) \equiv 0 \mod |WS|.$$

Let χ' be the character afforded by the $\mathbb{C}WS$-module V^H, so that by an easy representation theory, we have that

$$\chi'(gS) = \frac{1}{|S|} \sum_{h \in S} \chi(gh), \qquad gS \in WS,$$

and so χ' is also a real valued character. Thus in order to prove (1), we may assume that $S = 1$. Set $u_\chi(g) := u(V)(\langle g \rangle)$, then it has the value

$$u_\chi(g) = \operatorname{sgn} \langle \chi_{|\langle g \rangle}, 1_{\langle g \rangle} \rangle,$$

where $\langle\ ,\ \rangle$ stands for the inner product of characters. Now when $S = 1$, (1) becomes

(2) $$\sum_{g \in G} u_\chi(g) \equiv 0 \mod |G|.$$

In order to prove (2), it will suffice to show that u is a virtual character of G. In fact, we can show that

(3) $$u_\chi = (-1)^{\chi(1)} \det \chi,$$

where $\det \chi$ is the linear character of G defined by the composition

$$\det \chi : G \longrightarrow GL(V) \xrightarrow{\det} \mathbb{C}^*.$$

See Yoshida [Yo78]. In order to prove (3), we may assume that G is cyclic. Since $u_{\chi+\theta} = u_\chi u_\theta$ and $\det(\chi+\theta) = \det\chi\cdot\det\theta$, we may further assume that either χ is a real valued linear character or $\chi = \lambda + \bar{\lambda}$ for some nonreal linear character λ. In the first case, we have that $\langle \chi_{|\langle g\rangle}, 1_{\langle g\rangle}\rangle = 1$ if g is in the kernel of and $= 0$ otherwise, and so $u = -1$ or $+1$, respectively. Since $\det\chi = \chi$, (3) holds in this case. Next assume that $\chi = \lambda + \bar{\lambda}$, so that $\langle \chi_{|\langle g\rangle}, 1_{\langle g\rangle}\rangle = 0$ or 2 and $\det\chi = 1_G$. Thus (3) holds also in this case. The theorem is proved.

3. Some transfer theorems for the unit groups.

Let p be a prime. We put

$$\mathbb{Z}_{(p)} := \{ a/b \in \mathbb{Q} \mid a \in \mathbb{Z},\ b \in \mathbb{Z} - p\mathbb{Z} \},$$
$$A(G)_{(p)} := \mathbb{Z}_{(p)} \otimes_{\mathbb{Z}} A(G).$$

For a finite group H, the subgroup generated by all p'-elements of H is denoted by $O^p(H)$. When $O^p(Q) = Q$ (that is, Q has no normal subgroup of index p), the group

Q is called to be p-<u>perfect</u>. Let $Cl_p(G) \subseteq Cl(G)$ denote the classes of p-perfect subgroups.

There is a one-to-one correspondence between primitive idempotents of $A(G)_{(p)}$ and $Cl_p(G)$ (cf. [Di79] 1.4). An explicit formula of primitive idempotents was obtained by Gluck [Gl81] and Yoshida [Yo83]. Let μ be the Mobius function of the subgroup lattice of G and δ_G the function defined by

$$\delta_G(H, K) := \begin{cases} 1 & \text{if } H =_G K \\ 0 & \text{otherwise.} \end{cases}$$

Each primitive idempotent of $A(G)_{(p)}$ is then written in the form

$$e^p_{G,Q} = \sum_{(D)\in Cl(G)} \lambda_{G,Q}(D)\ [D \backslash G],$$

where $(Q) \in Cl_p(G)$ and

$$\lambda_{G,Q}(D) := \frac{1}{|N_G(D)|} \sum_{K \leq G} \mu(D, K)\ \delta_G(O^p(K), Q).$$

As a super class function, $e^p_{G,Q}$ has the value

$$e^p_{G,Q}(S) = \begin{cases} 1 & \text{if } O^p(S) =_G Q \\ 0 & \text{otherwise.} \end{cases}$$

For the finite group G, let $\underline{\underline{Set}}_f^{\,G}$ denote the category of finite (right) G-sets and G-maps. For two G-sets X and Y, let Y^X be the G-set consisting of all mappings of X to Y with G-action defined by $\alpha^g(x) := \alpha(xg^{-1})\cdot g$. For an element $a = [A] - [B]$ of $A(G)$, we furthermore define the exponential map

$$(-)^a : A(G)^* \longrightarrow A(G)^* \; ; \; u \longmapsto u^{A+B}.$$

We often write $u \uparrow a$ for u^a. By this action, $A(G)^*$ is an $A(G)_{(2)}$-module whose annihilator contains $2A(G)_{(2)}$.

Theorem B. (i) There is a decomposition

$$A(G)^* = \prod_{(Q)} A(G)^* \uparrow e^2_{G,Q},$$

where (Q) runs over $Cl_2(G)$, classes of 2-perfect subgroups.

(ii) Let Q be a 2-perfect subgroup of G and let P be a subgroup of $N := N_G(Q)$ such that P/Q is a Sylow 2-subgroup of N/Q. Then there are group isomorphisms:

$$A(G)^* \uparrow e^2_{G,Q} \cong A(N)^* \uparrow e^2_{N,Q} \cong (A(P)^*)^N \uparrow e^2_{P,Q}.$$

where the last group is the subgroup of $A(P)^* \uparrow e^P_{P,Q}$ consisting of all elements x such that

$$\mathrm{res}^P_{P^n \cap P}\, \mathrm{con}^n_P\, (x) = \mathrm{res}^P_{P^n \cap P}(x)$$

for any element n *of* N.

Theorem C. Let N *be a finite group with* 2-*perfect normal subgroup* Q. *Put* $W := N/Q$. *Let* $\bar{P} = P/Q$ *be a Sylow* 2-*subgroup of* W. *Then the following groups are isomorphic:*

(a) $A(N)^* \uparrow e^2_{N,Q}$,

(b) $\{\bar{u} \in A(W)^* \uparrow e^2_{W,1} \mid \bar{u}(S/Q) = 1 \text{ if } O^{2'}(S) \not\geq Q\}$,

(c) $\{\bar{v} \in A(\bar{P})^{*W} \mid \bar{v}(S/Q) = 1 \text{ if } O^{2'}(S) \not\geq Q\}$,

where $O^{2'}(S)$ *is the subgroup of* S *generated by all* 2-*elements and* $A(\bar{P})^{*W}$ *is the set of elements* v *of* $A(\bar{P})^*$ *such that*

$$\mathrm{res}_{\bar{P}^w \cap \bar{P}}\, \mathrm{con}^g(x) = \mathrm{res}_{\bar{P}^w \cap \bar{P}}(x) \quad \text{for all } w \text{ in } W.$$

Theorem D. Assume that the finite group G *has an abelian Sylow* 2-*subgroup. Let* Q *be a* 2-*perfect subgroup of* G. *Put* $N := N_G(Q)$, $W := N/Q$, *and let* P/Q *be an (abelian) Sylow* 2-*subgroup of* W. *Put* $L := N_G(P)$ $(\leq N)$, $\bar{L} := L/Q'$, $\bar{Q} := Q/Q'$, *and* $\bar{P} := P/Q'$, *where* Q' *is the intersection of subgroups of* Q *of odd prime index. Then the following hold :*

(i) $A(G)^* \uparrow e^2_{GQ} \cong A(\bar{L})^* \uparrow e^2_{\bar{L}\bar{Q}}$.

(ii) *If* Q *is perfect, then*

$$A(\bar{L})^* \uparrow e^2_{\overline{LQ}} \cong A(P/Q)^{*L/Q} \cong C_{(P/QP^2)}(L/Q),$$

where C *stands for the centralizer group*.

(iii) *Assume that* Q *is not perfect*. *If* $\bar{P}$ *is generated by elements* t *with* $C_{\bar{Q}}(t) \neq 1$, *then* $A(\bar{L})^* \uparrow e^2_{\overline{LQ}} = 1$, *and otherwise it is of order* 2.

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Appendix

(References on Burnside rings)

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